Dyscalculia:
Mathematics Learning Disability

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Agenda

1. Definitions of Dyscalculia
2. Risk Factors of Dyscalculia
3. Characteristics of Dyscalculia
4. Evidence-Based Mathematics Practices
Background

- Approximately 5% to 8% of school-age children exhibit mathematics learning disabilities (MLD; Badian, 1983; Murphy, Mazzocco, Hanich, & Early, 2007)

- Have persistent early difficulties in number sense (e.g., understanding number magnitude) and basic fact retrieval strategies (Geary, 2011).

- Have associated cognitive dysfunction (e.g., impairment of working memory and visuospatial skills)

- Approximately 20% to 60% of children with dyscalculia have co-occurrence or comorbid disorders (i.e., dyslexia or ADHD; Kaufmann & von Aster, 2012).
Questions

• What is dyscalculia?

• How can we identify children with dyscalculia?
Debates and Essential Features

• Different terminologies and criteria across different researchers and authorities
  - Dyscalculia or developmental dyscalculia
  - Specific learning disabilities in mathematics calculation or mathematics problem solving
  - Specific learning disorders with impairment in mathematics

• Neurodevelopmental disorder
• May first manifest during the years of formal schooling
• A life-long specific learning disability
Developmental Dyscalculia (Kosc, 1974)

- Czech researcher, Kosc, coined the term ‘developmental dyscalculia’ in 1974.

- Developmental dyscalculia is “a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder in those parts of the brain that are the anatomical-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions’.”
## Classification (DSM-V, ICD-11)

<table>
<thead>
<tr>
<th>DSM-V</th>
<th>ICD-11</th>
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</thead>
<tbody>
<tr>
<td>• Specific learning disorder</td>
<td>• Developmental learning disorder</td>
</tr>
<tr>
<td>• Symptoms persist for at least 6 months despite the provision of interventions</td>
<td>• Synonyms: Mathematics disorder, disorder of arithmetical skills</td>
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<tr>
<td>• A type of neurodevelopmental disorders</td>
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<tr>
<td>• With impairment in mathematics number sense, memorization of arithmetic facts, accurate or fluent calculation, and accurate math reasoning</td>
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<tr>
<td>• The affected academic skills are substantially and quantifiably below those expected for the individuals’ chronological age</td>
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<tr>
<td>• Impairment in mathematics is not due to a disorder of intellectual disabilities, sensory impairment, lack of availability of education, or lack of proficiency in the language of academic instruction.</td>
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*Note. DSM-V = Diagnostic and Statistical Manual of Mental Disorders, ICD = International Classification of Diseases.*
Individuals with Disabilities Education Act (IDEA, 2004)

- Definition of specific learning disability was included in previous versions of IDEA since 1975

- Specific learning disability means “a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations, including conditions such as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia.”
• **Does not achieve** adequately for the child’s age or to meet State-approved grade-level standards in mathematics calculation or mathematics problem solving.

• **Does not make sufficient progress** to meet age or State-approved grade-level standards in one or more areas when using a process based on the child’s response to scientific, research-based intervention.
Criteria for determining the existence of a specific LD

- The child does not achieve adequately for the child’s age or to meet State-approved grade-level standards in one or more of the following areas, when provided with learning experiences and instruction appropriate for the child’s age or State-approved grade-level standards:
  - Oral expression.
  - Listening comprehension.
  - Written expression.
  - Basic reading skills.
  - Reading fluency skills.
  - Reading comprehension.
  - Mathematics calculation.
  - Mathematics problem solving.
Procedure for the Identification of Specific LD

A State must adopt, consistent with 34 CFR 300.309, criteria for determining whether a child has a specific learning disability as defined in 34 CFR 300.8(c)(10). In addition, the criteria adopted by the State:

- Must not require the use of a severe discrepancy between intellectual ability and achievement for determining whether a child has a specific learning disability, as defined in 34 CFR 300.8(c)(10);
- Must permit the use of a process based on the child’s response to scientific, research-based intervention; and
- May permit the use of other alternative research-based procedures for determining whether a child has a specific learning disability, as defined in 34 CFR 300.8(c)(10).
Background

Previous reviews on the etiology and cognitive characteristics of MLD

Lack of studies on examining the mathematical performance beyond the cognitive characteristics

Examining developmental delays or maturational lags in mathematical and cognitive capacities

(Desoete & Roeyers, 2002)
Sources of Variability that Influence Academic Outcomes

(Fletcher, Lyon, Fuchs, & Barnes, 2007)
Neurobiological Factors: Brain Functioning

- Thinking initiation
- Reasoning
- Memory
- Speaking

- Knowing right from left
- Sensation
- Reading
- Understanding Spatial relationships

- Vision
- Color blindness

- Understanding language
- Behavior
- Memory, Hearing
Core Cognitive Processes

Cognitive Domain
- Working Memory
- Processing Speed
- Long-Term Memory
- Metacognition

Mathematical Domain
- Mathematical Calculation
- Word Problem Solving
- Arithmetic Strategies
- Number Sense
Working Memory Model (Baddeley and Hitch, 1974)

- Visuo-spatial scratch pad
- Central Executive
- Long-Term Memory
- Phonological Loop
  - Articulatory control
  - Phonological store
- Input
- Sensory memory
- Attention
- Decay
Working Memory (Baddeley and Hitch, 1974)

**Phonological Loop Working Memory**
- Responsible for the storage of verbal information (e.g., word, digit forward span, and word forward span)

**Visual-Spatial Working Memory**
- Responsible for the storage of mental images (e.g., visual matrix, picture sequence, and mapping and directions)

**Central Executive Working Memory**
- Coordinates and interacts with the above two working memory components (e.g., listening sentence span, digit/sentence span, backward digit span, and operation span)
Long-term Memory

- Declarative Knowledge: “I know that...even numbers end with the digits 0, 2, 4, 6, and 8!”
- Procedural Knowledge: “I know how...to pronounce and comprehend new vocabulary!”
- Episodic Knowledge: “I remember when...I graduated from high school!”
Information Processing Speed

• Processing relatively simple information quickly and easily, including counting speed, speed of retrieval

• Example: Speed of rapid naming of letters, numbers, and objects
Metacognition

- A deeper level of thinking that includes one’s ability to think about thinking
- How we understand, adapt, change, control, and use our thought processes.
- Example: Skills of prediction and evaluation
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How Fast Can you Count These Dots?
Which Number is Larger?
Previous Studies


Students with MLD had significantly limited performance on most mathematical and cognitive measures compared to students with No LD-age or grade.

- Metacognition (The least)
- Working Memory
- Information Processing
- Attention
- Cognitive Flexibility
• In most cognitive domains, students with LD and Students with ADHD demonstrated comparable performances.

• Students with LD and Students with ADHD had a significant difference on metacognition only.

• Students with LD showed significantly better metacognition than students with ADHD.
Math LD vs. Math and Reading LD

• Students with MLD had higher word problem solving and better arithmetic fact strategy than students with MLD/RLD.

• No significant group differences across all cognitive topics.
Math LD vs. Younger Children with no LD

• **Mixed findings** on mathematical and cognitive measures.

• **Working memory and metacognition could be subject to developmental delays** (Desoete & Roeyers, 2002).
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What Should We Teach Students?

Conceptual understanding VS. Procedural understanding
The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. $\frac{1}{2}$
D. 1

Explain your answer:

$$\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24} \text{ is closest to 20.}$$

(Petit, Laird, & Marsden, 2010)
What model represents the fraction below?

\[
\frac{4}{11}
\]
<table>
<thead>
<tr>
<th>Recommendation</th>
<th>Level of evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tier 1</strong></td>
<td></td>
</tr>
<tr>
<td>1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.</td>
<td>Moderate</td>
</tr>
<tr>
<td><strong>Tiers 2 and 3</strong></td>
<td></td>
</tr>
<tr>
<td>2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.</td>
<td>Low</td>
</tr>
<tr>
<td>3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.</td>
<td>Strong</td>
</tr>
<tr>
<td>4. Interventions should include instruction on solving word problems that is based on common underlying structures.</td>
<td>Strong</td>
</tr>
<tr>
<td>5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.</td>
<td>Moderate</td>
</tr>
<tr>
<td>6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.</td>
<td>Moderate</td>
</tr>
<tr>
<td>7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.</td>
<td>Low</td>
</tr>
<tr>
<td>8. Include motivational strategies in tier 2 and tier 3 interventions.</td>
<td>Low</td>
</tr>
</tbody>
</table>

Source: Authors’ compilation based on analysis described in text.
Effective Instructional Components

(Shin & Bryant, 2015)

1. Explicit, Systematic Instruction
2. Range and Sequence of Examples
3. Heuristic Strategies
4. Concrete and Visual Representations
1. Explicit, Systematic Instruction

Evidence: Strong
Sequence of Explicit, Systematic Instruction

- **Modeling**
  - Introducing new concepts
  - Teacher-led explicit modeling

- **Guided Practice**
  - Questioning *(Why? How?)*
  - Students’ verbalization

- **Independent Practice**
  - Self-evaluation
  - Cumulative, continuous review
2. Range and Sequence of Examples

Evidence: Strong
Vocabulary (Example vs. Nonexample)

- Learning mathematics as an acquisition of new words
  (Topping, Campbell, Douglas, & Smith, 2003)

(Texas Education Agency/University of Texas System, 2011)
CSA Instruction

- Gradually sequenced instruction
- Use of concrete and visual representations in conjunction with sequence of examples

Concrete
  - Manipulating physical objects

Semiconcrete
  - Drawing pictorial representation

Abstract
  - Writing mathematical symbols
<table>
<thead>
<tr>
<th>Concrete</th>
<th>Representational</th>
<th>Abstract</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Concrete" /></td>
<td><img src="image2.png" alt="Representational" /></td>
<td>$4 + 5 = 9$</td>
</tr>
</tbody>
</table>
3. Heuristic Strategies

Evidence: Strong
## Strategy Instruction

### Cognitive strategy instruction

- Teaching cognitive processes or routines
- Teaching meta-cognitive processes
- Self-regulation strategies *(SAY, ASK, CHECK)* *(Montague, 2008)*

### Schema-based instruction

- Identifying a word problem type
- Using a specific problem-type schematic diagram to solve the problem
- Organizing problem information *(Powell, 2011)*
Mnemonic Strategy (Cognitive Strategy)

- **STAR** strategies
  
  - **S**earch the word problem
  - **T**ranslate the words into an equation in picture form
  - **A**nswer the problem
  - **R**eview the solution
4. Concrete and Visual Representations

Evidence: Strong
Manipulative Instruction

- **Use of beans, construction paper to solve problems**

- **Perimeter**: Walking around the room; constructing a square on the geoboard; counting the distance from nail to nail

- **Area**: Counting room tiles; constructing four-by-four nails on the geoboard; dividing the square as many as one-by-one nail square
Perimeter vs. Area
Visual Representations

• Help students to better understand the meaning and relationship of fractions through relevant diagrams

• Example: Part-whole relationship is represented with a divided circle; multiplication of fractions is represented with a rectangular area model
Use of number line

- Extension of whole number concepts (Siegler et al., 2010)
Virtual Manipulatives

• **Interactive dynamic visual models** (e.g., pattern block, number line, vertical bar) that appear on a monitor and is intended to represent concrete manipulatives (Moyer, Bolyard, & Spikell, 2002)

• Web-based applications: Illuminations, Conceptua Math, Math Village, Smart Math, Fun Fraction

• Tablets: ABCya.com’s Virtual Manipulatives, Conceptua Math’s Student App, Illuminations
Interactive Visual Model Tools

- **Grouping:** Whole class, small group

  - Teacher modeling +
  - Active engagement

  - Exploring visual models +
  - Manipulating diagrams
THANK YOU

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